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**DYNAMICS OF DEFORMABLE MULTIBODY SYSTEMS USING  
RECURSIVE PROJECTION METHODS**

**Final Report**

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## ABSTRACT

In this investigation, generalized Newton-Euler equations are developed for deformable bodies that undergo large translational and rotational displacements. The configuration of the deformable body is identified using coupled sets of reference and elastic variables. The nonlinear generalized Newton-Euler equations are formulated in terms of a set of time invariant scalars and matrices that depend on the spatial coordinates as well as the assumed displacement field. These time-invariant quantities appear in the nonlinear terms that represent the dynamic coupling between the rigid body modes and the elastic deformation. A set of recursive kinematic equations, in which the absolute accelerations are expressed in terms of the joint and elastic accelerations are developed for several joint types. The recursive kinematic equations and the joint reaction relationships are combined with the generalized Newton-Euler equations in order to obtain a system of loosely coupled equations which have sparse matrix structure. Using matrix partitioning and recursive projection techniques based on optimal block factorization an order  $n$  solution for the system equations is obtained. The formulation developed in this research project allows for decoupling the joint and elastic acceleration equations without the need for finding the inverse or the LU factorization of large nonlinear matrices. The proposed new formulation is implemented on the digital computer in a fairly general way and the efficiency of the new algorithm is compared with other existing techniques. The techniques and computer programs developed in this research project are applied to the dynamic analysis and design of technological and industrial systems such as robotic manipulators and mechanism systems.

## 1. INTRODUCTION

Two approaches are commonly used in the study of the mechanics of multibody systems consisting of interconnected rigid bodies [1,2]. The first approach is based on Lagrange's equation where scalar quantities such as kinetic energy, potential energy and virtual work are used. Important concepts such as virtual displacements, generalized coordinates and generalized forces must be introduced in the Lagrangian dynamics. The second approach is based on Newtonian Mechanics wherein the motion of the rigid body in space is described by six scalar equations. The first three equations called Newton's equations describe the translation of the body and the remaining three equations which are called Euler's equations [3] describe the rotation of the body. In deriving Newton-Euler equations it is assumed that the origin of the rigid body reference is rigidly attached to the body center of mass. This assumption leads to the set of Newton-Euler equations in which the translation and rotation of the rigid body are dynamically decoupled. In the Newton-Euler approach, important concepts such as the angular velocity and angular acceleration must be introduced. The angular velocity and the angular accelerations can be defined in terms of the generalized rotational coordinates [4], and therefore Newton-Euler equations can be obtained using Lagrange's equation in a straightforward manner. Newton-Euler equations have been used in developing recursive dynamic formulations for open loop multibody systems consisting of interconnected rigid bodies [5]. The success of using this approach in multi-rigid body dynamics is mainly due to the fact that Newton-Euler equations are defined in a simple closed form.

Most existing recursive methods developed for the dynamic analysis of flexible multibody systems lead to a system of strongly coupled equations. Decoupling the joint and elastic

acceleration equations requires finding the inverse or the LU factorization of nonlinear matrices whose dimension depends on the number of elastic coordinates of the system. The difficulties which are encountered in decoupling the joint and elastic accelerations are mainly due to the sequence of the inertia projection used in most existing recursive methods.

Another problem of particular interest is the dynamic analysis of closed loop kinematic chains. Closed loops are handled by making cuts at selected secondary joints in order to form spanning tree structures. A set of algebraic constraint equations that describe the secondary joints can be formulated and augmented to the equations of motion of the open loop system in order to form the dynamic equations of the closed loop kinematic chains.

## **2. STATEMENT OF THE PROBLEM**

Several basic problems of multibody system design have been solved during the last few years. Most of the investigations were concerned with the dynamic behavior of multibody systems containing rigid links and accordingly the effect of elastic deformation on the dynamic behavior of multibody systems has not been fully investigated. Quite often, it is desirable to build light weight mechanical and structural systems which operate at very high speeds. The choice of the kinematic and dynamic parameters for the mechanical systems is greatly influenced by the flexibility of the system components.

It is the objective of this research project to develop, in a closed form, nonlinear generalized Newton-Euler equations for the dynamic analysis and control of mechanical systems that contain a set of interconnected deformable bodies each of which may undergo large relative translational and rotational displacements. A set of time invariant quantities that depend on the

assumed displacement field and are required to evaluate the terms that represent the nonlinear inertia coupling between the reference coordinates and the elastic deformation are identified for each deformable body. The identification of these invariants is crucial in our development since they provide a systematic approach to study the dynamics of deformable bodies and define the exact information required from a preprocessor structural dynamic program.

In this investigation, the kinematic relationships in which the absolute coordinates are expressed in terms of the joint relative coordinates are developed for several joint types. These equations with the generalized Newton-Euler equations can be used to obtain the system equations of motion in factored matrix form, which results in large systems of loosely coupled equations amenable to sparse matrix manipulation. Matrix partitioning and recursive projection techniques are then applied to symbolically lay out order  $n$  solution. The techniques proposed in this research can be applied to arbitrary open and closed loop kinematic systems to generate the necessary uncoupled equations.

A computational algorithm in which an appropriate link between finite element computer codes and dynamic analysis codes that employ matrix partitioning and recursive projection techniques is developed. In so doing, the dynamic analysis of open- and closed-loop deformable mechanical systems in terms of a coupled set of absolute, joint and elastic coordinates can be automated and carried out to the stage of numerical calculations in a fairly general way. The new computational algorithms is implemented on the digital computer and the efficiency of this algorithm is compared with other existing algorithms.

### 3. BRIEF OUTLINE OF RESEARCH FINDINGS

The objective of this research project was the development of computational methodology for the nonlinear dynamic analysis of articulated flexible mechanical systems. Most existing methods for flexible multibody dynamics lead to a set of acceleration equations in which the coefficient matrix is dense and highly nonlinear. Decoupling the relative joint and elastic accelerations in these methods require the inversion or the LU factorization of nonlinear matrices whose dimension depends on the number of elastic degrees of freedom of the system. Therefore, speaking of the order of an algorithm becomes meaningless since the number of elastic coordinates may vary from one body to another. A recursive projection method for the dynamic analysis of nonlinear articulated mechanical systems that consist of interconnected deformable bodies is developed. The absolute velocities and accelerations of the *leaf* or *child bodies* in the system are expressed in terms of the absolute velocities and accelerations of the *parent bodies* and the time derivatives of the relative joint coordinates. The dynamic differential equations of motion are developed for each body using the *generalized Newton-Euler equations*. The relationships between the actual joint reactions and the generalized forces combined with the kinematic relationships and the generalized Newton-Euler equations are used to develop a system of loosely coupled equations which has a sparse matrix structure. Optimal matrix permutation, partitioning, and projection methods are used to decouple the joint and elastic acceleration equations while maintaining the inertia coupling between the rigid body motion and the elastic deformation. Recursive projection methods are then applied in order to project the inertia of the leaf bodies onto their parent bodies. This leads to an optimal symbolic factorization which recursively yields the absolute and joint accelerations as well as the joint



reaction forces. This procedure does not require the use of Newton-Raphson algorithms in the numerical solution of the constrained dynamic equations of open-loop chains since the joint accelerations are readily available from the solution of the resulting reduced system of equations. Furthermore, the method requires only the inversion or decomposition of relatively small matrices and the numerical integration of a minimum number of coordinates.

In the analysis of closed kinematic chains, cuts are made at selected secondary joints in order to form spanning tree structures. Compatibility conditions and reaction force relationships at the secondary joints are adjoined to the equations of the open-loop systems in order to form the closed loop dynamic equations. Utilizing the sparse matrix structure of these equations and the fact that the joint reaction forces associated with elastic degrees of freedom do not represent independent variables, a procedure for decoupling the joint and elastic acceleration equations of closed loop mechanical systems was developed. Based on the *amalgamated formulation* developed in this research project, a computer program for the dynamic analysis of open and closed loop flexible mechanical systems was developed. In this computer program, the independent joint accelerations are integrated numerically using a direct numerical integration method that has a variable order and variable step size. The use of the numerical procedure developed in this research project was demonstrated using open loop and closed loop multibody mechanical systems.

During this research project several problems related to the dynamic formulations of flexible mechanical systems were examined. The research effort was focused on developing efficient methodologies for the computer aided dynamic analysis of open and closed loop flexible mechanical systems. The details of the analysis performed, the formulation developed, and the

applications considered in this research project are presented in the papers listed in section 4.

The tasks accomplished in this research project can be summarized as follows.

1. A spatial finite element formulation based on the generalized Newton-Euler equations was developed. This formulation accounts for all the terms that represent the nonlinear inertia coupling between the rigid body motion and the elastic deformation.
2. The recursive kinematic equations for mechanical systems that consist of interconnected deformable bodies were developed. The recursive equations were developed for the cylindrical, revolute, prismatic, and rigid joints.
3. The Newton-Euler equations, the recursive kinematic equations, and the joint reaction force equations are used to obtain a sparse system of loosely coupled equations. The solution of this system of equations defines the absolute, elastic, and joint accelerations as well as the joint reaction forces.
4. A procedure for decoupling the joint and elastic accelerations was developed. This procedure for eliminating the elastic accelerations leads to a system of equations whose dimension is independent of the number of the elastic coordinates of the system. This step for eliminating the elastic accelerations represent the first projection step.
5. An order  $n$  recursive projection algorithm was developed, where  $n$  is the number of bodies in the system. The order of the algorithm developed in this research project does not depend on the number of the elastic degrees of freedom of the system.
6. A recursive projection method was also developed for the analysis of closed loop kinematic chains. Closed loops are handled by making cuts at selected secondary joints.
7. The use of the methods developed in this research project were demonstrated using open and closed loop kinematic chains (Space crane, four bar mechanism).
8. The efficiency of the computer algorithms developed in this research project was compared with other existing methods. Significant saving in CPU time can be achieved as the result of decoupling the joint and elastic accelerations. Furthermore, the use of Newton-Raphson algorithms in the analysis of open loop kinematic chains is not required.
9. The effect of the coupling between the rigid body and elastic displacements was examined. The significant effect of the rigid body displacement on the wave

motion was demonstrated.

10. Several composite plate elements that can be used in the dynamics of spatial mechanical systems were developed. The use of four and eight noded composite plate elements in the analysis of spatial mechanisms was demonstrated.
11. Several graduate students received their Ph.D. and M.S. degrees during the last three years. The work of these students was closely related to the subject of the proposed research.
12. The results obtained in this research project were documented in several referred journal and conference publications.

#### **4. LIST OF PUBLICATIONS**

The research presented in the following journal papers is supported in full or in part by this project.

1. Gau, W.H., and Shabana, A.A., "Effect of the Finite Rotation on the Propagation of Elastic Waves in Constrained Mechanical Systems," ASME Journal of Mechanical Design, Vol. 114, No. 3, September 1992, pp. 384-393.
2. Jiang, J.J, Hsiao, C.L., and Shabana, A.A., "Calculation of Nonlinear Vibration of Rotating Beams by Using Tetrahedral and Solid Finite Elements," Journal of Sound and Vibration, Vol. 148 (2), 1991, pp. 193-213.
3. Shabana, A.A., "Constrained Motion of Deformable Bodies," International Journal for Numerical Methods in Engineering, Vol. 32 (8), December 1991, pp. 1813-1831.
4. Shabana, A.A., and Gau, W.H., "Propagation of Impact Induced Longitudinal Waves in Mechanical Systems with Variable Kinematic Structure," ASME Journal of Vibration and Acoustics, to appear.
5. Kirshner, F., Hsu, W.C., and Shabana, A., "Effect of the Order of the Finite Element and Selection of the Constrained Modes in Deformable Body Dynamics," Journal of Nonlinear Dynamics, Vol. 13, 1992, pp. 57-80.
6. Chen, D.C., and Shabana, A.A., "Dynamics of Initially Curved Plates in the Analysis of Spatial Flexible Mechanical Systems," Accepted for publication in the ASME Journal of Mechanical Design.

7. Palas, H., Hsu, W.C., and Shabana, A.A., "On the Use of the Momentum Balance and the Assumed Modes Method in the Transverse Impact Problems," ASME Journal of Vibration and Acoustics, Vol. 114, No. 3, July 1992, pp. 364-373.
8. Chen, D.C., and Shabana, A.A., "The Rotary Inertia Effect in the Large Reference Displacement Analysis of Initially Curved Plates," Accepted for publication in the Journal of Sound and Vibration.
9. Hwang, Y.L., and Shabana, A.A., "Dynamics of Flexible Multibody Space Cranes Using Recursive Projection Methods," International Journal of Computers & Structures, Vol. 43, No. 3, May 1992, pp. 549-563.
10. Shabana, A.A., Hwang, Y.L., and Wehage, R.A., "Projection Methods in Flexible Multibody Dynamics, Part I: Kinematics," International Journal for Numerical Methods in Engineering, Vol. 35, No. 10, December 1992, pp. 1927-1939.
11. Wehage, R.A., Shabana, A.A., and Hwang, Y.L., "Projection Methods in Flexible Multibody Dynamics, Part II: Dynamics and Recursive Projection Methods," International Journal for Numerical Methods in Engineering, Vol. 35, No. 10, December 1992, pp. 1941-1966.
12. Gofron, M., and Shabana, A., "Control Structure Interaction in the Nonlinear Analysis of Flexible Mechanical Systems," Accepted for publication in the Journal of Nonlinear Dynamics.
13. Shabana, A.A., and Hwang, Y.L., "Dynamic Coupling Between the Joint and Elastic Coordinates in Flexible Robot and Mechanism Systems," Accepted for publication in the International Journal of Robotics Research.
14. Kremer, J.M., Shabana, A.A., and Widera, G.E.O., "Large Reference Displacement Analysis of Composite Plates, Part I: Finite Element Formulation," Accepted for publication in the International Journal for Numerical Methods in Engineering.
15. Kremer, J.M., Shabana, A.A., and Widera, G.E.O., "Large Reference Displacement Analysis of Composite Plates, Part II: Computer Implementation," Accepted for publication in the International Journal for Numerical Methods in Engineering.
16. Hwang, Y.L., and Shabana, A.A., "Decoupled Joint-Elastic Coordinate Formulation for the Analysis of Closed Loop Flexible Multibody Systems," Submitted to the ASME Journal of Mechanical Design.

The following are the conference publications:

1. Gau, W.H., and Shabana, A.A., "Effect of the Finite Rotation on the Speed of Propagation of Impact-Induced Waves in Constrained Elastic Systems," presented at the Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Blacksburg, Virginia, June 25-27, 1990.
2. Gau, W.H., and Shabana, A.A., "Effect of the Finite Rotation on the Propagation of Elastic Waves in Constrained Mechanical Systems," Proceedings of the 21st Biennial Mechanism Conference, DE-Vol. 24, Chicago, Illinois, September 16-19, 1990.
3. Cetinkunt, S., Shabana, A.A., and Hwang, Y.L., "Dynamics and Motion Control of Space Cranes," Presented at the 7th International Conference on Advanced Science and Technology, Argonne National Laboratory, IL, March 30, 1991.
4. Chen, D.C., and Shabana, A.A., "Effect of the Coupling Between Bending and Extension on the Large Displacement Analysis of Initially Curved Plates," Proceedings of the Fourth International Conference on Recent Advances in Structural Dynamics, Southampton, England, July 15-18, 1991.
5. Hwang, Y.L., and Shabana, A.A., "On the Solution for the Accelerations Using Different Multibody Dynamics Formulations," Presented at the First U.S. National Congress on Computational Mechanics, Chicago, IL, July 21-24, 1991.
6. Chen, D.C., Shabana, A.A., and Rismantab-Sany, J., "Generalized and Actual Constraint Forces in the Inverse Dynamic Analysis of Spatial Flexible Mechanical Systems," Proceedings of the ASME 1991 Design Automation Conference, Miami, FL, September 1991.
7. Hwang, Y.L., and Shabana, A., "Decoupling the Joint and Elastic Accelerations in Flexible Multibody Vehicle Dynamics," Proceedings of the 1992 Canadian Society of Mechanical Engineering CSME FORUM 1992, Montreal, Canada, June 1-5, 1992.
8. Gofron, M., and Shabana, A.A., "Effect of the Linearization of the Coriolis and Centrifugal Forces on the Feedforward Control Law of Flexible Mechanical Systems," Presented at the Fourth Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Blacksburg, Virginia, June 7-11, 1992.
9. Kremer, J.M., Shabana, A.A., and Widera, G.E.O., "Dynamic and Stress Analysis of Spatial Flexible Mechanical Systems," Proceedings of the 22nd ASME Biennial Mechanisms Conference, DE-Vol. 47, pp. 573-584, Scottsdale, Arizona, September 13-16, 1992.

## 5. LIST OF PARTICIPATING SCIENTIFIC PERSONNEL

Several graduate research assistants have contributed directly or indirectly to the completion of this project. Their names are

1. Dr. Wei-Hsin Gau, received his Ph.D. in the Winter of 1991.
2. Dr. Da Chi Chen, received his Ph.D. in the Spring of 1991.
3. Dr. John Kremer, received his Ph.D. in the Fall of 1991.
4. Dr. Wei-Chen Hsu, received his Ph.D. in the Spring of 1992.
5. Dr. Yunn-Lin Hwang, received his Ph.D. in the Spring of 1992.
6. Mr. M.K. Sarwar, currently working on his Ph.D.
7. Mr. J.H. Choi, currently working on his Ph.D.

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4. Kane, T.R., Likins, P.W., and Levinson, D.A., "Spacecraft Dynamics," McGraw-Hill, 1983.
5. Luh, J.Y.S., Walker, M.W., and Paul, R.P.C., "On-Line Computational Scheme for Mechanical Manipulators," ASME Journal of Dynamic Systems Measurement and Control, 1980, Vol. 102, No. 2, pp. 69-76.

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